Assignment I Math 2010 Prof. Y. Kwong

1/. Given $\vec{u}, \vec{v} \in V_3$ such that $\vec{u}, \vec{v} \neq \vec{0}$ ($\vec{0} = \langle 0,0,0\rangle$), prove
(i) $|\vec{v}|\vec{u} + |\vec{u}|\vec{v}$ and $|\vec{v}|\vec{u} - |\vec{u}|\vec{v}$ are perpendicular to each other.
(ii) Let $\vec{w} = \left(\frac{|\vec{v}|}{|\vec{u}|+|\vec{v}|}\right)\vec{u} + \left(\frac{|\vec{u}|}{|\vec{u}|+|\vec{v}|}\right)\vec{v}$, show that $\vec{u}$ and $\vec{v}$ make equal angle with $\vec{w}$.

2/. For any $\vec{u}$ and $\vec{v}$, prove the followings:
(i) $\vec{u}\cdot\vec{v} = \frac{1}{4}\left(|\vec{u}+\vec{v}|^2 - |\vec{u}-\vec{v}|^2\right)$
(ii) $|\vec{u}\times\vec{v}|^2 = |\vec{u}|^2|\vec{v}|^2 - (\vec{u}\cdot\vec{v})^2$

3/. Prove the following inequalities:
(i) $|\vec{u}\cdot\vec{v}| \le |\vec{u}||\vec{v}|$ (Cauchy Schwartz Inequality)
(ii) $|\vec{u}+\vec{v}| \le |\vec{u}|+|\vec{v}|$ (Δ-Inequality)
(iii) Suppose we generalize it to the space of vectors in $\mathbb{R}^N$, $N \geq 3$. Let $\vec{u} = \langle u_1, \cdots, u_N\rangle$ and $\langle v_1, \cdots, v_N\rangle$, how would you prove (i) in this case.

4/. Given $P(1,1,0)$, $Q(1,0,4)$ and $R(0,2,5)$, find the area of ΔPQR.

5/. Prove that $P(2,0,0)$, $Q(4,2,-2)$, $R(-5,0,4)$ and $S(4,-5,1)$ are coplanar.

6/. Given a plane $Ax+By+Cz+D=0$ in space and let $P(x_0,y_0,z_0)$ be any point in space. Derive a formula for computing the perpendicular distance of P to the plane. (Hint: look at the 2-D case).

7/. Find non-zero vectors $\vec{u}, \vec{v}$ and $\vec{w}$ such that $\vec{u}\times\vec{v} = \vec{u}\times\vec{w}$ but $\vec{v}\neq\vec{w}$.

8/. Find the equation of the plane through the line
$$\frac{x+1}{3} = \frac{y-1}{2} = \frac{z-2}{4}$$ which is also $\perp$ to the plane $2x+y-3z+4=0$.

9/. Prove that $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})$ could be expressed as a linear combination of $\{\vec{a},\vec{b}\}$ as well as a linear combination of $\{\vec{c},\vec{d}\}$

10/. Let L_1, L_2 be 2 lines whose equations are respectively given by $L_1: \frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and $L_2: \frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$.
Prove that L_1 and L_2 are skew iff $\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} \neq 0$.